On-line Computation of MTIE using Binary Decomposition and Direct Search with Sequential Data Reducing

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***Abstract*—In this paper the methods enabling assessment of Maximum Time Interval Error (MTIE) in the real-time are presented and compared. The idea of real-time quasi-parallel computation of MTIE is described first. Then two different methods enabling real-time MTIE calculation are described. The results of experimental tests performed for different data sequences and using different computers are presented and discussed.**

I. INTRODUCTION

Maximum Time Interval Error (MTIE) is used for describing some aspects of quality of telecommunication network timing signals [1, 2, 3]. The MTIE point estimate is commonly computed for a series of observation intervals using the sequence of time error samples measured at some network interface. The computation usually follows the time error measurement process. Because of rather long time of MTIE plain computation, several time effective methods enabling rather fast MTIE computation were proposed [4, 5, 6], some of them by the authors of this paper [7, 8, 9].

Some features of the methods (sequential data reduction, long window's shifts) enabled to formulate of the idea of on-line MTIE computation, which is performed in the real time, during the measurement of time error samples, and parallel for several observation intervals [8]. The method proposed in [8], called extreme fix with sequential data reducing (EFSDR), brought very good experimental results regarding time efficiency. Unfortunately, the application of this method may result in incorrect MTIE estimates. Because of the way of data reduction some sample values, which affect the MTIE value, may be omitted for specific arrangement of samples and windows' (observation intervals) lengths [9]. As result, the MTIE value may be underestimated. Two next methods oriented at the real-time quasi-parallel MTIE assessment, bringing good results regarding time efficiency as well as computation correctness, were described and experimental tested by the authors of this paper in [10]. One of the methods (called direct search with sequential data reducing, DSDR) uses sequential data reduction, as EFSDR method, but the way of reduction (being result of direct search of the extremes) enables to avoid the omitting of significant data. Second method was adopted for real-time calculation time effective extreme fix method [7] without data reduction. Both methods, as well as EFSDR method are data-dependent methods of MTIE assessment, i.e. their mechanisms of windows' review and data reduction depends on the data values. Therefore their time efficiency depends on the data behavior. In this paper the application of the binary decomposition for the real-time calculation is presented. This non-data-dependent method, proposed by Bregni and Maccabruni [6], uses binary decomposition of the time error sequence for data reduction in the calculation process. The decomposition is performed independent of the data values, and therefore the time efficiency of the method does not depend on the data behavior.

In order to calculate the MTIE estimate simultaneously for several observation intervals in the real time, all necessary operations should be performed in the time period between two sampling instants, i.e. during the sampling interval τ_0. The ability of real time assessment depends on the following conditions: number and length of the observation intervals considered, computational power of the measurement equipment, and time error data behavior (in the case of data-dependent algorithm). In the paper the results of experimental comparison of the data-dependent and non-data-dependent methods are presented. The calculations were performed for several different data sequences taken with sampling interval τ_0=1/30 s, which is often used in the telecommunication applications. The results of calculation using personal computers with different processors and clock's frequency are presented and compared.

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II. Maximum Time Interval Error

The point estimate of the Maximum Time Interval Error is defined in the international standards as the maximum peak-to-peak time error variation of a given timing signal, with respect to an ideal timing signal within a particular time period [1, 2, 3]. If the results of time error function measurements $x(t)$ take the form of N equally spaced samples $\{x_i\}$, MTIE can be estimated from the formula

$$M\hat{T}IE(n\tau_0) = \max_{1\le k\le N-n}\left(\max_{k\le i\le k+n} x_i - \min_{k\le i\le k+n} x_i\right) \qquad (1)$$

where $\{x_i\}$ is a sequence of N samples of time error function $x(t)$ taken with sampling interval τ_0, $\tau=n\tau_0$ is an observation interval, and n=1, 2, ..., N–1.

Following directly the formula (1) in order to find the estimate of MTIE for the observation interval τ, all intervals having the width of τ, existing in the sequence of N time error samples, must be reviewed. The window having the width of $\tau=n\tau_0$ and including n+1 samples is set at the beginning of the data sequence $\{x_i\}$ and then it is shifted with the step of τ_0 to the end of the sequence. For each window's location the peak-to-peak value of time error in the window is found. The maximum peak-to-peak value found for all existing locations of the window is the value of MTIE(τ) estimate. The process of window reviewing does not depend on the data value. The complexity of calculation grows with n and therefore the direct method is really time-consuming. The idea of direct search (plain computation) of MTIE is presented in Fig. 1.

The time effective methods of MTIE assessment were proposed in order to avoid the time-consuming plain computation. Several different mechanisms were applied to reduce the complexity of computation. In the process of the MTIE search using the extreme fix (EF) method some window's locations are excluded from inspection if the peak-to-peak value for each of these locations is not greater than the value found until now, or if a greater peak-to-peak value may be found for the successive window's locations [7]. The EF method is based on fixing the positions of minimum and maximum samples for a given window's location. The general rule is that the next window's location is originated at the first extreme previously found.

The methods with sequential data reduction were dedicated especially for the MTIE assessment for a series of observation intervals, starting from some τ_{min} until some τ_{max}. The methods follow the suggestion according to which during the MTIE search process for some observation interval τ_i ($\tau_i>\tau_{min}$) we find the extreme samples for some window's location from the set of extreme samples found previously during the MTIE search for the smaller observation interval τ_{i-1} ($\tau_i>\tau_{i-1}$). Only these samples may influence the MTIE value for the observation interval τ_i. Other time error samples in the data sequence do not matter in the MTIE search process. Therefore, we can reduce the number of time error samples used for the MTIE calculation.

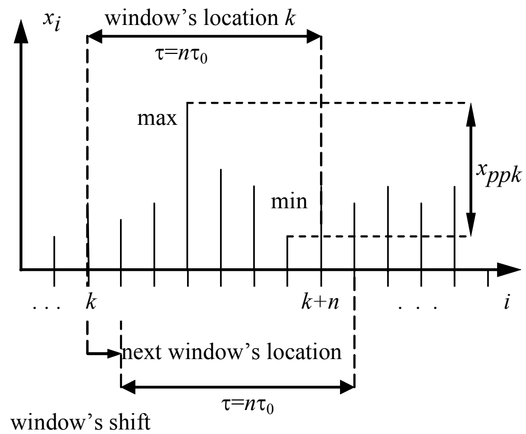

Figure 1. The idea of direct search for MTIE

Two methods with sequential data reduction were proposed by the authors of this paper. The first method, called EFSDR, uses extreme fix search for the raw data sequence as well as for the reduced data [8]. Unfortunately, it may produce errors and the final MTIE value may be underestimated. In order to avoid the errors, the rules of the extreme searching were changed. The second method – direct search with sequential data reduction (DSDR) – uses plain computation at each level of the procedure (for raw and reduced data) [9].

Another method was proposed by Bregni and Maccabruni in [6]. This method uses binary decomposition of the data sequence. At the first step of the calculation process the 2-samples windows are considered. Two neighboring samples are compared and the maximum and minimum values for each pair are selected. The maximum of the difference between them within one window is the MTIE for the 2-samples observation interval. At the second step previously selected extreme values are used for finding the maximum and minimum for 4-samples windows (the comparison of the extremes for the neighboring pairs is performed only). Then the MTIE is searched. At the successive steps the next windows (containing increasing number of samples) are considered by the creation from the previously analyzed windows, and the data reduction process proceeds. The method with binary decomposition is characterized by the limitation: the lengths of observation intervals considered should be a power of 2.

III. Real time MTIE calculation

The formula of the MTIE estimator allows to perform the calculation of the parameter in the real-time, during the time error measurement. In this way we can observe the value of the parameter during the long lasting measurement process. Any possible wrong behavior of the analyzed signal (recognized, if MTIE exceeds the limit) enables applying proper activity of a maintenance team. Therefore, there is no necessity to wait until the two processes – the measurement followed by the parameter's computation – are completed.

A general procedure of the real-time quasi-parallel MTIE calculation for a series of observation intervals is as follows:

1. Measure a new time error sample.
2. Compare the new sample with current maximum and minimum.
3. If current window's location is filled out with samples, fix the extremes for this location.
4. Check if the current window's location is filled out with samples for the next longer observation interval.
5. If so, find the extremes for this location and check the conditions for the next longer observation interval.
   If no, measure a new sample.
6. When the measurement is finished, continue the computation for the remaining locations of each longer observation interval.

The choice of the algorithm suited for the real-time parallel calculations is very important. If we want to calculate the MTIE estimate simultaneously for several observation intervals, all necessary operations should be performed in the time period between two successive sampling instants, i.e. during the sampling interval τ_0. Therefore, the calculation algorithm should be time effective in order not to exceed the sampling interval. Two methods of the real-time MTIE calculation were proposed by the authors of this paper: direct search with sequential data reduction and extreme fix search independent for each observation interval [9, 10]. These methods were detailed described in [9, 10] and only the main principles will be presented below.

The real-time computation using DSDR method for the first (shortest) observation interval τ_{min} begins with the first measured time error sample. Each new sample measured is compared with current maximum and minimum values, until the first window's location is filled out by the samples. Then the extreme values for this location are fixed. Each successive measured sample creates a new window's location. The extreme samples found for each window's location create new data sequences with reduced number of items. The items of reduced data sequences are used for the MTIE estimate calculation for the observation intervals longer than τ_{min}. The first location of the next longer window is not analyzed until all samples situated in this location are reviewed by the preceding window. The example of computation using DSDR method is presented in Fig. 2. In the case of real-time calculation using EF method, the computation procedures for each observation interval run independently. Window's locations of longer observations intervals are analyzed after filling out by the samples without waiting for the analysis by the preceding shorter windows. All windows are activated after filling out their first locations by the samples. The extremes found for some observation interval do not affect the calculation process for other observation intervals.

Similar as the EF and DSDR methods, the binary decomposition method can be adopted for the real-time MTIE computation. The computation starts with the first sample measured. Each successive measured sample is compared with the sample measured previously. The result of the comparison – pair of maximum and minimum – is stored for further analysis. The difference between these values (peak-to-peak value for 2-samples window) is compared with current maximum peak-to-peak value (maximum peak-to-peak found until now). Next, if the sufficient number of samples was already measured, the analysis for the 4-samples window is performed. The current result of 2-samples window analysis is compared with the result obtained two sampling intervals earlier – extreme values found for the preceding neighboring 2-samples window. The result of current 4-samples window analysis is stored for further 8-samples window analysis. Next windows having power of two samples are analyzed (if sufficient number of samples was measured) using the previously stored results of analysis of smaller windows. All these operations should be performed within the time interval between two successive time error samples.

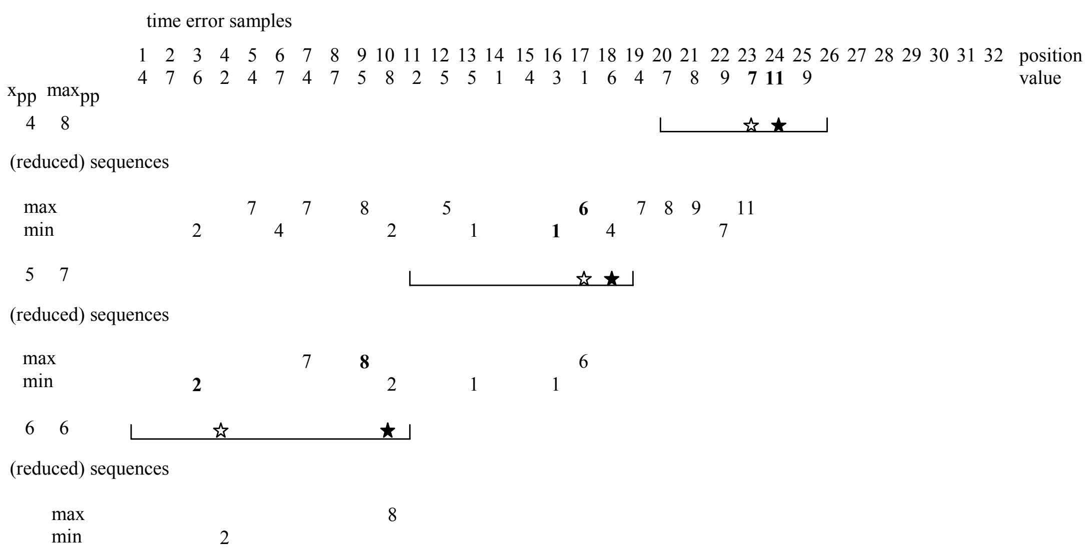

Figure 2. The example of real-time MTIE computation using direct search with sequential data reduction for observation intervals having 6, 8 and 10 samples – middle stage of the measurement process

The example of real-time MTIE calculation using binary decomposition is presented in Fig. 3-5. The early stage of the process is presented in Fig. 3. Four time error samples were measured until now. Fourth sample is compared with third sample (2-samples window analysis) and the result of the analysis pair of maximum – 6 and minimum – 2 is stored. The difference between these values – 4 is currently maximum peak-to-peak value for 2-sample observation intervals. Currently obtained pair of extreme values for 2-samples window is used next for the analysis of the first 4-samples window's location. This pair (6 and 2) is compared with the pair (7 and 4) being the result of the first 2-samples window analysis. As result, the pair of 7 and 2 is stored for further operations, and the difference between these samples is current maximum peak-to-peak value. The middle stage of the measurement and computation process is presented in Fig. 4. Sixteen time error samples were measured until now. Sample number 16 is compared with the sample number 15 and the pair of 4 and 3 is stored as result of 2-samples window analysis. This pair is compared next with the pair obtained in the fourteenth step (5 and 1) being the result of analysis of the samples number 13 and 15. As result of the 4-samples window analysis, the pair of 5 and 1 is stored. This pair is compared next with the pair obtained in the 12th step (8 and 2) being the result of the analysis of the 4-samples window covering the samples from number 9 till 12. As result we have the pair of 8 and 1, which is the pair of extreme values for the 8-samples window covering the samples from number 9 till 16. Because 16 samples were measured until now, the first location of 16-samples window can be analyzed in this step. The pair of 8 and 1 (just obtained for 8-samples window) is compared with the pair obtained for 8-samples window in the step number 8. The result – pair of 8 and 1 – is the pair of maximum and minimum for the first location of 16-samples window. The peak-to-peak values are computed and compared for appropriate observation intervals: 2, 4, and 8-samples windows. The final stage of the computation and measurement process is presented in Fig. 5. The time error measurement is finished. All partial results – pairs of extreme values – are stored. The MTIE values for each observation interval considered are known directly after the measurement is finished.

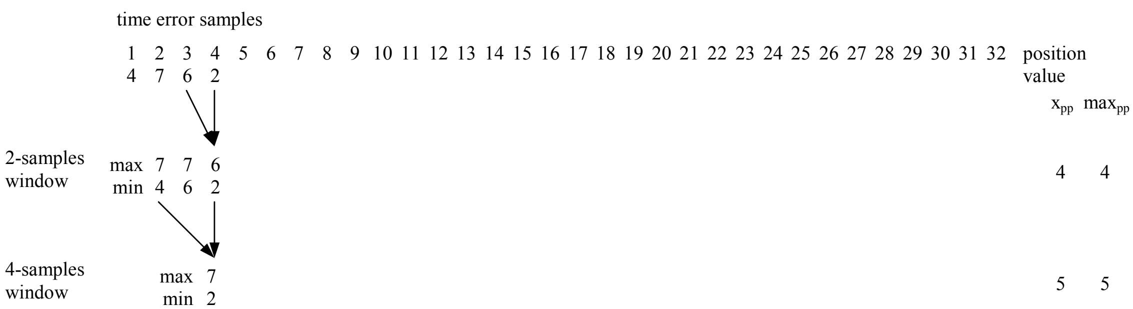

Figure 3. Real-time MTIE computation using binary decomposition – early stage of the measurement process, three locations of 2-samples window and one location of 4-samples window were analyzed

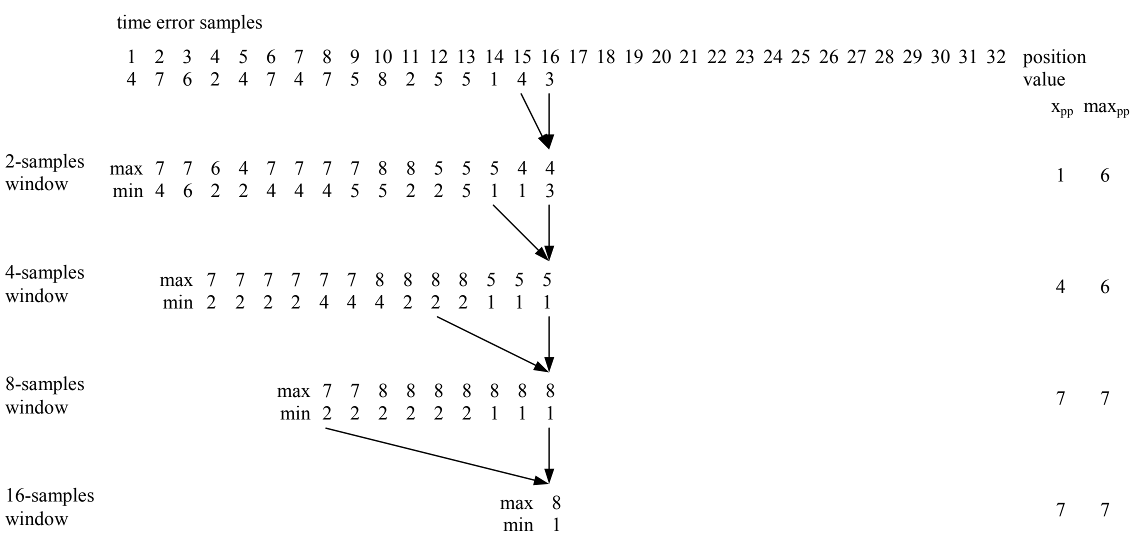

Figure 4. Real-time MTIE computation using binary decomposition – middle stage of the measurement process

time error samples

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 position
4 7 6 2 4 7 4 7 5 8 2 5 5 1 4 3 1 6 4 7 8 9 7 11 9 12 8 9 6 8 5 7 value

end of measurement

2-samples window
max 7 7 6 4 7 7 7 7 8 8 5 5 5 4 4 3 6 6 7 8 9 9 11 11 12 12 9 9 8 8 7
min 4 6 2 2 4 4 4 5 5 2 2 5 1 1 3 1 1 4 4 7 8 7 7 9 9 8 8 6 6 5 5
MTIE(τ_0)=6

4-samples window
max 7 7 7 7 7 7 8 8 8 8 5 5 5 4 6 6 7 8 9 9 11 11 12 12 12 12 9 9 8
min 2 2 2 2 4 4 4 2 2 2 1 1 1 1 1 1 1 4 4 7 7 7 7 8 8 6 6 5 5
MTIE($3\tau_0$)=6

8-samples window
max 7 7 8 8 8 8 8 8 8 8 6 6 7 8 9 9 11 11 12 12 12 12 12 12 12
min 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 4 4 7 7 6 6 5 5
MTIE($7\tau_0$)=10

16-samples window
max 8 8 8 8 8 8 9 9 11 11 12 12 12 12 12 12 12
min 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
MTIE($15\tau_0$)=11

Figure 5. Real-time MTIE computation using binary decomposition – end of the measurement, final MTIE values are known.

IV. Calculation experiment

The results of the experimental tests of the real-time MTIE computation using EF and DSDR methods were presented in [10]. The MTIE values were computed using both methods for the series of observation intervals arranged in the logarithmic scale between 0.1 s and 1000 s, starting from 0.1 s. The results have showed very good behavior of the EF method for small number of observation intervals (up to 10) from the range 0.1-10 s. For longer observation intervals analyzed simultaneously and their greater ranges, the results using DSDR method were much better.

In this paper the results of experimental tests and comparison of DSDR method and binary decomposition method are presented. The calculations were performed off-line but the on-line work was imitated. Three different time error sequences were used. The first time error sequence represents one of typical noises of the timing signals – white phase modulation (WPM). The second sequence was obtained from the comparison of two different GPS disciplined oscillators (Fig. 6). The third sequence (denoted as MSG, Fig. 7) was obtained from the measurement of the signal generated by the internal oscillator of some measurement system. The time error samples were taken with the sampling interval τ_0=1/30 s during the period of 4 000 s. The length of the time error sequences is 120 001 samples.

Because we cannot compute the MTIE using binary decomposition for arbitrary chosen observation intervals, the calculations were performed for changing numbers of observation intervals having the power of 2 samples. The starting (smallest) observation interval was 2-samples interval for binary decomposition and 4-samples interval for DSDR method (the computation for 2-samples interval is not necessary in this case). The maximum observation interval was changed and took the values of 32 samples (which corresponds with 1.03 seconds), 512 samples (17.03 s), 4096 samples (1136.5 s), and 32768 samples (1092.2 s). Therefore we have performed the computation for 4, 8, 11, and 14 simultaneously analyzed observation intervals using DSDR methods, and for 5, 9, 12, and 15 intervals using binary decomposition.

Two personal computers with Pentium IV 1.4 GHz and Pentium IV 3.0 GHz were used in the experimental tests. The observed quantity was the maximum time spent for calculation for sampling interval. We have assumed that this time cannot exceed the length of sampling interval τ_0=1/30 s = 0.0333… s.

The time of calculation using DSDR method is presented in Table I (for the computer with Pentium IV 1.4 GHz microprocessor) and Table II (for the computer with Pentium IV 3.0 GHz microprocessor). The maximum time of operations performed for one sampling interval does not exceed the sampling interval in the case of WPM and GPS time error sequences for all ranges of observation interval. In the case of MSG sequence (showing monotonic change of time error process, caused by the difference between frequencies of the compared oscillators) the maximum time exceeds the limit (1/30 s) for computation performed for 11 and 14 intervals using both computers.

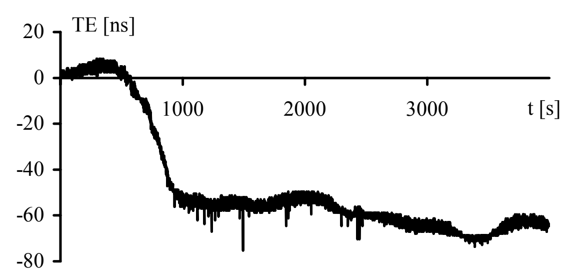

Figure 6. GPS time error sequence

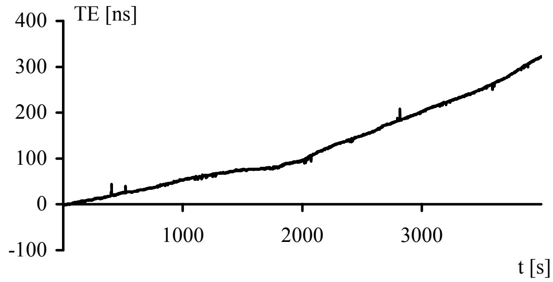

Figure 7. MSG time error sequence

The time of calculation using binary decomposition is presented in Table III (for the computer with Pentium IV 1.4 GHz microprocessor) and Table IV (for the computer with Pentium IV 3.0 GHz microprocessor). The maximum time of operations performed for one sampling interval does not exceed the sampling interval in the case of each time error samples. The operations performed for one sampling interval using binary decomposition was less time consuming than using DSDR method, especially for the MSG time error sequence and for greater ranges of observation intervals.

Surprisingly enough the application of the computer with higher clock frequency does not improve the results obtained respectively. It is because the operations of reading, writing and comparison dominate in the computation procedures using both methods. The reduced samples are stored in the data files on the computer's hard drive, and the time of access to the data plays important role.

TABLE I. TIME OF CALCULATION USING DSDR METHOD FOR COMPUTER WITH PENTIUM IV 1.4 GHZ

Number of intervals	Range of intervals [s]	WPM	GPS	MSG
		t-max [s]	t-max [s]	t-max [s]
4		0.0017	0.0017	0.0017
8		0.0028	0.0044	0.0082
11		0.0038	0.0066	0.0401
14		0.0044	0.0137	0.0566

TABLE II. TIME OF CALCULATION USING DSDR METHOD FOR COMPUTER WITH PENTIUM IV 3 GHZ

Number of intervals	Range of intervals [s]	WPM	GPS	MSG
		t-max [s]	t-max [s]	t-max [s]
4		0.0014	0.0016	0.0017
8		0.0027	0.0041	0.0082
11		0.0038	0.0061	0.0379
14		0.0044	0.0134	0.0543

TABLE III. TIME OF CALCULATION USING BINARY DECOMPOSITION METHOD FOR COMPUTER WITH PENTIUM IV 1.4 GHZ

Number of intervals	Range of intervals [s]	WPM	GPS	MSG
		t-max [s]	t-max [s]	t-max [s]
5		0.0011	0.0011	0.0011
9		0.0017	0.0017	0.0017
12		0.0022	0.0022	0.0022
15		0.0028	0.0028	0.0028

TABLE IV. TIME OF CALCULATION USING BINARY DECOMPOSITION METHOD FOR COMPUTER WITH PENTIUM IV 3 GHZ

Number of intervals	Range of intervals [s]	WPM	GPS	MSG
		t-max [s]	t-max [s]	t-max [s]
5		0.0011	0.0011	0.0011
9		0.0016	0.0016	0.0017
12		0.0019	0.0022	0.0019
15		0.0024	0.0025	0.0025

V. CONCLUSIONS

Both methods described in the paper enable real-time quasi-parallel computation of MTIE. The regularly running sequential data reduction using binary decomposition results in better behavior in comparison with the DSDR method, especially for the widest range of observation intervals and monotonic changes of data. The required dimensions of observation intervals (power of 2) limit the application of the binary decomposition method. The reduced data sequences obtained during DSDR real-time computation can be used off-line, after the end of measurement process, e.g. for MTIE calculation for any arbitrary observation interval value not considered during on-line analysis. Using binary decomposition we cannot compute the MTIE value for arbitrary observation interval, or for chosen number of observation intervals within a specific range, if their number of samples is not power of 2.

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